Economics as Distributed Computation

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Summary: In human societies diverse people act purposively with powerful but limited cognitive processes, interacting directly with one another through technologically-facilitated and physically-mediated social networks. Agent-based computational modeling takes these features of humanity—behavioral heterogeneity, bounded rationality, network interactions—at face value, using modern object-oriented programming techniques to create agent populations that have a high degree of verisimilitude with actual populations. This contrasts with mathematical social science, where fantastic assumptions render models so cartoon-like as to beg credibility—stipulations like identical agents (or a single 'representative' agent), omniscient agents (who accurately speculate about other agents), Nash equilibrium (macro-equilibrium arising from agent-level equilibrium) and even the denial of direct agent-agent interaction (as in general equilibrium theory, where individuals interact only with a metaphorical auctioneer). There is a close connection between agent computing in the positive social sciences and distributed computation in computer science, in which individual processors have heterogeneous information that they compute with and then communicate to other processors. Successful distributed computation yields coherent computation across processors. When such distributed computations are executed by distinct software objects instead of physical processors we have distributed artificial intelligence. When the actions of each object can be interpreted as in its 'self interest' we then have multi-agent systems, an emerging sub-field of computer science. Viewing human society as a large-scale distributed system for the production of individual welfare leads naturally to agent computing. Indeed, it is argued that agents are the only way for social scientists to effectively harness exponential growth in computational capabilities.

Keywords: economics, distributed computation, multi-agent systems

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1 Introduction: The Economy as a Distributed System

Consider the following social situation. There are a large number of individuals interacting through their regular social networks, each engaged in purposive (i.e., self-interested) behavior. The network connects each of the individuals to at least a few others, but no one is connected to all others. The individuals receive periodic communication from those with whom they are connected, but there may be significant delays in the transmission of such messages. Each individual is capable of reasoning about and acting on the information it receives, but no agent can build a complete internal model of all other individuals, nor forecast the exact nature of the messages it will receive in the future. Overall, at the group or population level, the myriad actions by the many individuals accrete into macro-level activities that may be meaningful in and of themselves, such as when the individuals engage in collective action. Macro-level activity then feeds back to the individuals, causing each of them to adjust its behavior in some way.

Written at such a high level of abstraction, this 'social situation' has many interpretations. It might represent the internal workings of a firm, where the individuals are workers who communicate information on the progress of production or the design of next year's product. In this case the result of all the individual actions are goods having economic value. If such products bring handsome sums in the marketplace then the workers may not alter their behavior in any significant way in subsequent periods. Alternatively, if the products find few buyers then the workers will use such feedback to modify their activity or else perish economically. In a different interpreation, the interacting individuals might represent a market, where objects are being traded between the agents, each of whom has some forecast for how the market will unfold over the short or long run. Each individual may have somewhat different information on which it bases its prediction, and one role of the market *qua* market is to aggregate these diverse forecasts to come up with a clearing price, i.e., a price at which no individual wishes to engage in further trade. Or not—it may be that there are a number of so-called technical traders (sometimes called noise traders) in the market who believe—maybe erroneously, maybe not—that certain patterns in prices exist and can be exploited. The existence of such traders may so corrupt the price aggregation function of the market that the actual prices emanating from it deviate significantly from information aggregation ('efficient markets') prices.

Although I have called the abstract situation above 'social,' it need not involve humans, nor any form of biological life for that matter. At the very generic level described above the interacting individuals could just as well be interacting computers, inter-connected on a network, each perhaps with an explicit task to perform, contingent on the receipt of data. Overall, at the level of the whole network, the computers may be generating new and novel results, this despite the fact that no single computer has any information on the global goal. Of course, it could be that global information is made available to the individual processors, and this alters their behavior subsequently. It is easy to imagine that this macro-level information may either improve or degrade the performance of the individuals.

The main point of this interpretive exercise has been to suggest that there is a close relation between the modern conception of distributed computation, on the one hand, and economics specifically, and social science generally, on the other.

Recent papers by computer scientists argue for interpreting "Computation as Economics" (Huberman and Hogg 1995, Huberman 1998). These papers describe a variety of uses of economic ideas within computer science (CS), primarily within the sub-fields of artificial intelligence (AI) that have come to be known as distributed artificial intelligence (DAI) and multi-agent systems (MAS). While there is, undeniably, increasing use made of economic principles within CS, the core of CS has little or nothing to do with economics. Automata theory, databases, programming, algorithms, these are the central ideas of the CS curriculum today and a typical undergraduate student will wade through these disparate topics and never encounter economics in any significant way.[1] Indeed, CS today is much closer to engineering than to economics, even with respect to its intrinsic notion of efficiency. That is, computational efficiency refers to the number of operations needed to arrive at a satisfactory answer, with efficient algorithms requiring fewer operations, much as an efficient steam engine is one that makes effective use of its fuel. These notions of efficiency are utterly unlike economic efficiency, which by convention means Pareto efficiency and refers to the welfare effects of rearranging economic resources between agents within an economy.

Here we shall espouse the reverse of Huberman, and argue that there are important ways in which it is meaningful to speak of "Economics as Computation" and more generally of social science as a form of computer science. That is, there are principles from CS, especially in the areas of (1) distributed computation (Barbosa 1996), (2) object-oriented programming (OOP), and (3) multi-agent systems (Weiss 1999), that provide a solid foundation upon which a *modern* science of economics can be built.[2] For such computational tools provide the technology for relaxing the unrealistic assumptions of the reigning neoclassical synthesis, through the use of recent advances in computing.

This paper, then, is an implicit argument for the *sufficiency* of the multi-agent approach: modern computing is sufficient for the creation of a more powerful social science. A further thesis is *necessity*, that the only feasible way to harness modern computer technology for progress in the *positive* social sciences is to utilize multi-agent systems technology in the creation of models capable of reproducing social phenomena. Such model building efforts are at the heart of scientific explanation, as typified by Cartwright (1983): “To explain a phenomenon is to

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[1] For example, within AI the text of Winston (1992) makes essentially no mention of economics, although the more recent introduction to the subject by Russell and Norvig (1994) does emphasize the 'agent' as the principal unit of analysis, and so takes on a more game theoretic flavor.

[2] It is also possible to situate economics within the formal theory of computation (cf. Velupillai 2000), although I agree with Simon (1978) who long ago noted that computational complexity considerations seem more relevant to economic theory than automata-theoretic ones.

find a model that fits it into the basic framework of the theory and thus allows us to derive analogues for the messy and complicated phenomenological laws which are true of it…[T]he success of the model depends on how much and how precisely it can replicate what goes on."

2 The Computational Architecture of Economies

Human societies consist of diverse individuals, each with significant but bounded cognitive capabilities, distributed over space and within social networks, who interact directly with one another and with naturally-occurring and man-made objects. These individuals are essentially purposive in their actions, behaving in neither perfectly rational nor completely random ways. They act in their own self-interest and in accord with group norms and conventions.[3] Each individual accumulates over its lifetime significant knowledge concerning both the natural and social worlds. Important aspects of overall societal knowledge is held in common and collected in books and other media that can outlive individuals. But significant portions of the sum total of humanity's wisdom is not socially stored and is only imperfectly communicated, because it is both highly distributed and tacit in character (Hayek 1937, Polanyi 1958).

Societies function through the ongoing, decentralized interactions of physically heterogeneous and cognitively diverse individuals. Each person is more or less adaptive, never fully-optimizing, gleaning data from its environment and experimenting with alternative actions in order to inductively determine how to behave in new or unusual situations. Each person builds mental models both of its physical surroundings and the individuals with whom it interacts. These mental models often have a causal and dynamic character, e.g., 'if I do this then person X will think that.' In essence, people conduct mental simulations of their worlds (Davies and Stone, 1995). The data used in such mental models is always more or less out of date, such as when one bases an action purely on past interactions and has no way to determine whether the arrival of new information has altered the behavior of the person to whom the action is directed. Nor are such data necessarily consistent—indeed, larger amounts of data may hold conflicting information. In toto, a society is a large-scale, highly distributed network of agents, each of whom engages is continual real-time mental simulation of its immediate physical and social worlds. Societies of agents conduct these mental simulations in parallel, with some actions highly synchronized and others occurring asynchronously. The extent to which these myriad parallel, distributed thoughts and actions aggregate into coherent structures at the social level determines the overall character and performance of a society. Understanding the conditions under which specific societal

[3] Indeed, it is conventional to call self-destructive behavior pathological, and to label people who act in complete disregard of others *sociopaths* (Aaron 1994).

characteristics emerge constitutes an important component of the enterprise of social science.

It is conventional modeling practice in the methodologically individualist social sciences to specify the behavior of agents and then deduce the aggregate consequences of such behavior. Usually there is one or at most a few distinct types of agent behavior specified, so that all agents execute the same behavioral rules, e.g., utility maximization. However, each agent's internal states may be unique (e.g., preferences, endowments), thus permitting agent behavior to be heterogeneous across the population.

The class of parallel computing systems where each processor has the same instructions but heterogeneous data are called 'single instruction, multiple data' or SIMD. The advantage of this architecture computationally is that since all processors have the same code they can all execute one cycle of instructions in the same amount of physical time, meaning the processors are operating synchronously. It is tempting to think that multi-agent systems might naturally be implemented as SIMD, in order to take advantage of specialized high performance hardware, even if there is no necessity for perfect synchronization. Examples of SIMD hardware include digital signal processors (DSPs) and the cellular automata machine (CAM). While SIMD architectures have been used for certain physical and biological models, such as pattern formation and forest fire models, they have not been much utilized for social science modeling. For the perfect synchrony in such hardware is at best an imperfect representation of the timing of human social interactions, and at worst a fatally flawed assumption that impresses systematic artifacts into the resulting models. Indeed, human societies are very imperfectly synchronized (Huberman and Glance 1993, Axtell 2001).[4]

That real societies are asynchronous will seem so second nature to many that any further argument for such a depiction may seem overwrought. But it is important to remember that the norm in the mathematical theory of dynamical systems is to have each component of the system update synchronously (Luenberger 1979). The conventional way to deal with time lags mathematically is through so-called delay equations, but this doesn't alter the essentially synchronous character of dynamical systems theory.

However, from the theory of distributed computing a general mathematical formalism applicable to the decentralized social world of human interactions can be formulated through the partially asynchronous, parallel model of computation (Bertsekas and Tsitsiklis 1993). Here we interpret this formalism in the context of multi-agent systems. We shall find that by systematically incorporating out-of-date and stochastically arriving information into expressions for the evolution of agent populations we can, depending on the exact nature of such information lags, obtain results that are quite different from ones formulated under synchronous updating assumptions.

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[4] In contrast, many important biological processes rely crucially on synchronization (Nowak and May, 1992).

2.1 Mathematics of Distributed Social Interactions

Consider a population of N agents, each on whom has both internal states, representing its values, aspirations, memories, intentions, and beliefs about other agents, for example, as well as partially observable external states, e.g., its endowments. For the i^{th} agent, call its vector of states, $x_i \in X_i$, a Euclidean space, say, having dimension n_i, i.e.,

$$X_i \subseteq \mathbb{R}^{n_i} .$$

The dimension of agent states can vary across the population, with the overall state having dimension n, i.e.,

$$n \equiv \sum_i n_i ,$$

and the overall state space noted by X,

$$X \equiv X_1 \times X_2 \times \cdots \times X_N .$$

Define $x(t)$ as the state of the agent population at time t,

$$x(t) \equiv \left(x_1(t), x_2(t), \ldots x_n(t)\right) \in X, x_i \in X_i .$$

In general, there is some overlap between the states of distinct agents. That is, certain elements of X_i will also be in X_j , $j \neq i$, such as when two agents each have information on the magnitude of a stock market index, say, or the local weather. However, the values the agents have for the variable need not be the same, because they are generally out-of-date by some different amount, due to asynchronous updating. Overlap in agent state vectors also arises because agent have beliefs about other agents—their states, past actions, intentions, beliefs, and so on, which can also be substantially out of date.

Agents update their states asynchronously. T^i is the set of times when x_i is updated, and the set of all such update times is defined by $T \equiv \{ T^1, T^2, \ldots, T^n\}$. For each agent i there are n_i variables

$$\tau_j^i(t) \le t, j \in \left\{1, \ldots, n_i\right\}$$

which describe the age of the current information about the j^{th} component when i updates at time $t \in T^i$. For many components the information will not be significantly out of date, such as an agent's memories of past actions. A specific set T together with all the τ define a *scenario* or, more informally, a *run*.

Each agent's rules of behavior are specified by a function, $f_i: (X, t) \to X_i$. This function could be the result of some individual utility maximization calculus, the result of a production decision, or any other decision process. In practice, the domain of this function will not be the entire state space, but rather will be restricted to the set of other agents with whom the agent interacts, i.e., its social network. This social network can, in principle, evolve over time, which is one reason for the explicit time dependence of f_i. The individual agent dynamics then unfold according to

$$x_i(t+1) = x_i(t) \quad \forall t \notin T^i \tag{1}$$

$$x_i(t+1) = f_i\left(x_1\left(\tau_1^i(t)\right), x_2\left(\tau_2^i(t)\right), \ldots, x_n\left(\tau_n^i(t)\right)\right) \quad \forall t \in T^i . \tag{2}$$

The system as a whole advances through time according to

$$x(t+1) = f(x(\tau), t) . \tag{3}$$

We wish to consider only those agents who are actually interacting with one another, so in lieu of full asynchronism, which places no limits on how out of date the agent information can be, we will instead utilize a partially asynchronous specification. This means that there is some time, M—call it the societal memory—beyond which no individual has information on previous states. Stated slightly differently, the society is purged all information that is M periods old, i.e.,

$$t - M < \tau_j^i(t) \le t .$$

With this notation in place it is possible to state several properties of such parallel, distributed , asynchronous agent interactions.

Call $z(t)$ the concatenation of the M most recent periods of state information,

$$z(t) = \left(x(t), x(t-1), \ldots, x(t-M)\right) .$$

The behavior of the agent system (3)—for example, its convergence properties—are then described with respect to $z(t)$. Assume that the set of fixed points of f is not empty. This might be established by the Leray-Schauder-Tychonoff theorem, for example. Call x^* such a point, i.e.,

$$x^* = f\left(x^*\right)$$

with z^* the M period fixed point. It is possible to establish conditions under which this fixed point will be achieved. These results have the general character that either (a) convergence obtains for any value of the system memory, M, as long as M is finite, or (b) convergence occurs only if M is sufficiently small.

The convergence results obtain by analogy with the Lyapunov theorem from dynamical systems theory (Luenberger 1979). If there exists a function, d: $Z \to [0, \infty)$, such that

$$d(z(t+1)) < d(z(t)),$$

then every limit point of the iteration (3) yields the fixed point z^*. This result guarantees convergence to fixed points under quite general conditions. The main difficulty in applying this method is determining an appropriate d.

While this formulation of distributed computing is somewhat messier, notationally, than the corresponding formulation of synchronous updating, it is much more plausible as a description of real human behavior. Now, if it turned out that the system memory, M, played little or no role in the analysis then it could usefully be neglected and we would recover conventional dynamical formulations. However, as alluded to above, this is not the case in general. Indeed, rather than consider distributed computing as an inferior version of serial computing, it turns out that convergence can actually be faster under asynchronous conditions (Bertsekas and Tsitsiklis 1993). Furthermore, certain iterative processes that fail to converge synchronously may do so asynchronously, so there is even a sense in which this messier world can be advantageous.

Let us interpret the meaning of this formalism. First, it says that agents are not automatons who spend all their time engaged in social interaction. Rather, agents spend significant time doing things we do not model, occasionally waking up to interact socially. When they do this, they use the current information they have, even though much of it is likely old, in order to decide what to do. Certain aspects of each such social interaction are communicated through the society of agents, but this propagation of information is both time consuming and not guaranteed to reach all agents. As new data arrives to each agent it incorporates it into its future decisions, throwing out information that is older than M periods. If social equilibria exist—say driving on the right side of the road, to mention a coordination problem—then under mild conditions such distributed execution of an agent system will converge to one (of perhaps several) fixed points.

Compare this approach with the conventional 'social planner problem' of neoclassical economics, in which a single omniscient and benevolent social planner globally maximizes societal utility. That the solution to this optimal control problem is identical to Arrow-Debreu general equilibrium is seen as a strength of these approaches (Nordhaus 1992). The decentralized, distributed formulation of the problem given above admits no such identity, suggesting that the ostensibly decentralized general equilibrium caricature of equilibrium is highly unrealistic.

2.2 Distributed Exchange

To be more concrete, consider the application of this formalism to the problem of pure exchange. Specifically, there is a population of agents, each of whom has strictly convex preferences, say, and positive endowments of all goods. When agents are activate they are permitted to make welfare-improving trades with other agents at mutually agreeable prices. Interpreted as a decentralized exchange process, e.g., bilateral exchange, this formulation clearly reaches a Pareto optimal configuration. But as the trading process proceeds over time prices change, so that although the exchange path is individually rational the end state can display significant wealth effects. That is, while Walras' law is satisfied at each round of trading, it fails over the whole course of trades since prices are changing (Axtell 2002). This process cleaves the agent population into two classes of agents, those that gain wealth from the exchange process and those that lose it.

More can be said about exchange under such conditions. Consider a population having equal endowments but heterogeneous preferences. Whether such endowments are valued at the first price or the last (market-clearing) one, the initial wealth in the population will be Dirac distributed. But the subsequent process of exchange generates price dispersion and leads to the production of horizontal inequality between the agents.

Compare this process with Walrasian tâtonnement, where the market-clearing prices are first computed and then the agents engage in decentralized exchange at these prices. Interestingly, in certain conditions this requires a greater number of agent-agent interactions to produce an approximately Pareto optimal result than does the exchange process with local prices. That is, price dispersion can *improve* convergence to an equilibrum. Walrasian exchange, known to have high complexity in the abstract (Papadimitriou 1994) ends up being computationally less efficient than a model in which agents act myopically—the number of interactions required to equilibrate the decentralized market can be fewer than the number necessary to equilibrate the Walrasian market.

Finally, the statement in Nordhaus (1992) that the two distinct formulations of general equilibrium are governed by the same equations is analogous to a methodology common in computer science, that of reducing one problem to another. However, such demonstrations of equivalence have nothing to do with whether or not the basic problem is tractable. Indeed, reductions of this type preserve computational complexity, so demonstrating formal equivalence is a way of proving the computational intractability of conventional formulations of Walrasian general equilibrium models.

Formal modeling in the social sciences has the character of SIMD, as we have alluded to above. Certain models, such as discrete time dynamical systems models in capital theory are indeed perfectly synchronized. In other models agents are activated at random, so the models are asynchronous. Clearly the real social world, while partially synchronous, is also asynchronous in many important ways. In agent-based models both synchronous and asynchronous activation regimes have been studied and compared (Axtell 2001). The great flexibility of agent modeling with regard to activation regime is a powerful feature of this modeling approach.

3 Agents as (Distributed) Objects

There are a variety of ways to implement such models computationally, given the highly distributed, decentralized character of agent interactions, Older programming languages as well as specialized mathematical software have sufficed for the creation of agent models.[5] One way to represent agents in such systems is to instantiate a vector for each agent attribute, the dimensions of which are the size of the agent population. One then accesses agent i by getting the i^{th} element of each attribute vector. A somewhat more compact representation is to bring all the agent

[5] An incomplete list of languages and frameworks employed in social science research includes Ascape (Inchiosa and Parker 2002), BASIC (Holland 1996), C (Jones *et al*. 1997), C++ (Tesfatsion 2002), Excel (Krugman 1996), Gauss (Lux 1998), Java (), Lisp (Danielson 1996), Mathematica (Gaylord and D'Andria 1997), MatLab (LeBaron 1999), Objective C (Arthur *et al*. 1996), Pascal (Axelrod 1997), RePast (Cederman 1999), SDML (Moss *et al*. 1998), SmallTalk and SWARM (Luna and Staffanson, 2000).

attributes together so that the entire population is a single matrix, and agent *i*'s attributes can be accessed as a row (or column) of the matrix. With this representation there is a definite sense in which each agent is a contiguous chunk of address space, even though this will not (usually) be a contiguous physical memory space. A further refinement in representation is obtained by recourse to a specialized 'agent' data structure of the modeler's specification—a 'struct' in C or a 'record' in Pascal. Here each agent's data is physically represented contiguously in memory (modulo boundaries).

This increasing abstraction reaches its highest form in the object model—object oriented programming (OOP)—in which data and methods for modifying the data are brought together. This paradigm, pervasive in modern CS, is a very natural way to implement agents. It facilitates essentially any interaction structure (social network) and activation regime. This technology is especially powerful in the case of agents having identical behavioral rules. For here one need only program the methods of a typical agent once and then the corresponding methods for all agents are available. Increasing the number of agents in a model is then merely a matter of letting the operating system allocate the program more memory, for no additional programming is required.[6] Thus, agent computing has a 'small source, large execution' character.

Once an agent is conceived of and implemented as an object it is a relatively easy matter today to distribute the agent objects across multiple machines on a network. Various protocols exist for managing such distribution over networks as vast as the entire internet itself. In the future, large-scale models may be 'run' in just this fashion. But just how large must a model be in order to necessitate distributed computing? This is the subject we turn to in the next section.

4 Harnessing Moore's Law for Progress in Economics

Agent-based computational modeling was not feasible a generation ago, and barely possible a decade ago.[7] It is the dramatic progress in computer technology that has made multi-agent modeling possible. This era of revolutionary technical change[8] that we all are living through is especially apparent when one looks at the main competitor of multi-agent models, i.e., mathematical theorizing.

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[6] In practice, there is such performance advantage to keeping all agents in memory that the size of an agent population one can feasibly work with will often be limited by one's development environment.

[7] Microsimulation, which began in the late 1950s (e.g., Orcutt *et al.* 1961), is similar in spirit ot agents but quite different in character. While it also argues for a decentralized perspective, its formulation of household level supply and demand equations makes it one level of aggregation less distributed than agent systems.

[8] See Nordhaus (2000) for a history of this revolution in performance.

Progress in mathematics has been regular and significant over the last decades. Nonlinear dynamics, chaos, complexity, these are just a very few of the many areas of mathematics where important progress has been made. Indeed, according to some we live in a mathematical 'golden age,' in which a growing population of well-trained mathematicians is provided sufficient resources to conduct fruitful research of the highest quality. But the central mathematical tools employed by social scientists have not really undergone this same revolution. Given the focus on equilibrium in economics, theorems on the existence of fixed points have been workhorses. However, since mechanisms by which such fixed points can be achieved are rarely postulated, these results are of vague relevance empirically. This naturally leads to dynamics.

While ideas from nonlinear dynamics have percolated into economics and other social sciences, the relatively low dimensional nature of such models casts doubt on their ultimate relevance given the vast size and scale of real economic and social systems. Furthermore, technical requirements of such models are so severe as to essentially never be met in practice—time invariance of parameters, initial conditions that are knowable to high precision and so on.

Stochastic dynamics are an intermediate point mathematically between these two islands—fixed points on the one hand, full-blown dynamics on the other. But such models introduce important new problems. For example, detailed knowledge of the nature of the underlying stochasticity is crucial to fully understand the long run properties of the models.[9] But the extent to which such processes are 'colored' or unbiased in reality is poorly understood. It is not the case that random 'accidents' are the main driving forces of real social history, so we shouldn't expect them to be determinant in stochastic games, for example (Goeree, Holt 1997). "People make their own history" as Marx famously wrote, they don't merely live it waiting to see where exogenous shocks will take them.

Conventional economic theory is essentially based on writing out agent behavior mathematically, such as first order conditions derived from rationality assumptions, or as equilibrium conditions resulting from no arbitrage postulates. Subsequently these specifications are solved via operations research techniques. Overall, the way in which modern computing power is utilized in economics is remarkably similar to the way early digital computers were first used—e.g., to solve equations numerically (Judd 1999), to generate random numbers. These methods are derivative of similar techniques developed in the physical sciences over the past 40 years (Mirowski 2001). Limiting computational economics to such numerical workouts is both backward-looking from the perspective of economic methodology and, even more problematically, fails to make any significant use of modern advances in computing hardware and software.

For there is just no way to fill up a gigabyte of RAM with equations governing either representative agents or aggregations of homogeneous agents. Disk drives having 100 gigabyte capacity can hold many times over the results of millions of regressions (e.g., Salai-Martin 1998). Faster clock speeds mean less waiting for

[9] See Bergin and Lippman (1996).

complicated integrations, and lead one to believe that the 'curse of dimensionality' can at least be held at bay for 'realistically-sized' problems. More capable processors turn formerly large problems into small ones, as when one's answer appears immediately after hitting the 'RETURN' key instead of after many minutes. But the only satisfactory way to fully utilize the vast performance increases in CPU speed and memory density is through agents.

It is Moore's law that is responsible for such dramatic changes in the computational landscape over the past generation. Essentially, this law states that the power of digital computation grows exponentially. Data on the evolution of transistors/CPU are shown in Figure 1; for similar data see Moravec (1990), Kurzweil (2000) and Nordhaus (2002).

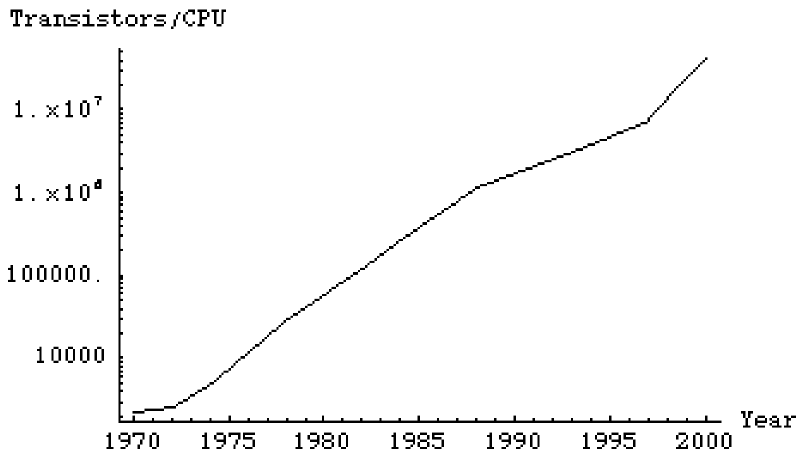

Fig. 1. Moore's law realized (Intel 2001; see also Moravec, 1990 and Kurzweil, 2000)

Since the ordinate is in logarithmic coordinates, specify that the number of transistors/CPU(t) = $k2^{t/T}$, were t is calendar time, T is the time constant of growth, and k is a constant. Transforming both sides of this equation results in ln(transistors/CPU(t)) = kt ln(2)/T. Since the data are nearly linear, estimating T means rearranging and computing the slope as T = (2000-1970) ln(2)/(ln(4.2x10^7)-ln(2.25x10^3)) ~ 2. Thus, every 2 years the number of transistors in new CPU designs double. This vast expansion of CPU complexity over time is responsible for the concomitant increase in performance of microcomputers. Memory densities have experienced similar growth and hard disk densities have had their growth recently accelerated further.

All this means that we can build larger and larger agent models. My experience in creating the Sugarscape (Epstein and Axtell 1996) and subsequent models is shown in Table 1, where the number of agents feasibly modeled on then state-of-the-art workstations is shown. These models were all created in relatively low level languages (e.g., Pascal, C and C++).

Table 1. Growth of agent capabilities on single workstations, native source code in C++

Model	Years	Agents
Sugarscape (Epstein and Axtell 1996)	1992-1996	$O(10^2)$ - $O(10^3)$
Artificial Anasazi (Axtell *et al.* 2002)	1995-2000	$O(10^2)$ - $O(10^4)$
Distributed Exchange (Axtell 2002)	1997-present	$O(10^2)$ - $O(10^6)$
Multi-Agent Firms (Axtell 1999)	1998-present	$O(10^3)$ - $O(10^7)$
Emergent Cities (Axtell and Florida 2002)	2000-present	$O(10^5)$ - $O(10^7)$

The progress evidenced in Table 1 leads one to speculate that the foreseeable future will bring substantially larger and potentially much more interesting models into the realm of feasibility.

Larger models promise more than mere quantity, for greater internal complexity can arise as size increases.[10] In the main, as human societies grow in size they also grow more complex, as one sees in comparing the relatively simple society of Black Mesa (Axtell *et al.* 2002) with the highly structured one of Chaco Canyon in the American Southwest (Gumerman . Tens of thousands of Native Americans inhabited Chaco at its peak, a number feasibly modeled today.

Looking beyond such anthropological models, once we learn more about how to 'grow' human social institutions, including realistic markets, firms having internal governance structure, governments, and so on, one can tentatively forecast the sizes of agent models that will be feasible over the coming decade. I offer one such forecast in Table 2.

Table 2. Prospective growth of agent capabilities on single workstations

Model	Feasible by	Agents
Chaco Canyon	present	$O(10^4)$ - $O(10^5)$
Economic Sector	2004-2006?	$O(10^5)$ - $O(10^6)$
Small Economy	2006-2008?	$O(10^6)$ - $O(10^7)$
Large Economy	2008-2010?	$O(10^7)$ - $O(10^8)$

Writing code in a platform independent environment, e.g., Java, knocks the maximum number of agents back by perhaps as much as an order of magnitude, delaying these capabilities by 5 - 8 years.

Certainly this is a great time to be alive if you are a user of high performance computing. Numerical economics simply does not have the wherewithal to fully utilize these revolutionary forces. Agents seem to be the only way to effectively harness exponentially growing computing power in the service of economics specifically and social science in general.[11] Furthermore, building agent models leads

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[10] This point was made in physics a generation ago by Anderson (1973).

[11] A recent article Joy (2000) argues that we need to worry about the growing power of our computing machines in the sense that once they become sentient they will quickly become superior to us, intellectually, physically, and ultimately perhaps economically and politically. However, purely on organizational grounds this is highly unlikely: no matter

naturally to relaxation of neoclassical modeling assumptions, a subject explored in the next section.

5 Agent Computing for Generalizing Economic Theory

A canonical usage of agents in economics is becoming clear: specify agent state variables and rules of behavior, let the system spin forward in time and observe the emergent macrostructure. This usage poses an immediate challenge to all varieties of mainstream theory, for situating behaviorally plausible agents in realistic interaction environments may or may not yield neoclassical results. By testing the robustness of conventional models, agents play a powerful disciplinary role, delimiting credible models from implausible ones.

For there are a large number of reasons why seemingly plausible theorems might fail in practice—e.g., perhaps a theorem only obtains asymptotically but the agent model, like the real world, runs in finite time; maybe noise, omnipresent in agent models, is sufficient to destabilize the equilibria that are known to exist as a consequence of a theorem; it could be that a theorem is sensitive to the agent interaction structure, but analytical results are available only for unrealistic social networks (e.g., lattices, random graphs); or conceivably a theorem is brittle to generalization along multiple dimensions simultaneously (e.g., altered interaction topology and activation regime).

But the most likely reason why most theorems are likely to fail once equivalent models are implemented with agents and progressively generalized is due to the heroic assumptions commonly employed in mathematical theorizing.

5.1 Societal Equilibrium Requires Agent Equilibrium?

It is either a tacit or explicit claim of game theorists (Binmore 1987, 1988) that explanation of social patterns and regularities at the aggregate level demand agent-level equilibrium, typically Nash or some refinement.[12] Clearly, if equilibrium at the agent level obtains there results macro-level equilibrium. But agent level equilibrium is not a necessary condition for aggregate equilibrium, merely a sufficient

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how intelligent individual machines become, they would find it difficult to organize themselves socially. For such intelligent forms would have no history of highly evolved self-governance structures, as do humans, and almost certainly would not be well-served by the kinds of institutions that humans have prospered under over the past several millennia—crudely, the family, private property and the state. In short, Joy's idea is naive from a social science perspective, a kind of negative utopia in the spirit of the literary genre that includes George Orwell, Stanislaw Lem and Philip K. Dick.

[12] Against this position see (Gilboa and Matsui 1991) and (Shibik 1999).

one (Axtell 1992).[13] A different path to such configurations is via adaptive behavior within large agent populations, such that the macro-state is stationary.

To make this more concrete, consider an agent model of firm formation (Axtell 1999). Here, individual agents pursue utility improvements non-cooperatively in team production environments. At the end of each period of the model, each team's production is tallied and various compensation rules allocate income to the agents. Each time an agent is activated it may stay a part of its current team, migrate to another team, selected at random, say, or it can start-up a new team, depending on which option it believes will yield the greatest welfare. While Nash equilibria always exist in this strategic environment, they turn out to be dynamically unstable for sufficiently large teams. Operationally, this just means that there is no way to partition the agent population into teams that is individually rational. What is observed in the model is the formation of productive teams that have a life of their own, admitting new employees while continually shedding other agents as outside opportunities arise. This is not unlike the real-world, where job turnover is a constant feature of labor markets. However, despite this continual flux and adaptation at the agent level, there emerges in this model stationary aggregate distributions of firm size and output, firm growth rate, and firm lifetime. In the agent population, while the fortunes of individual agents are somewhat variable, depending on how well their firm is doing at any instant, the distribution of income also assumes a stationary, highly skew configuration.

Under what conditions are claims about agent-level equilibrium reasonable, and when are they unlikely to be realized in practice? While this is a difficult question in general, one clear cut case *against* agent equilibrium occurs when the Brouwer or Kakutani fixed point theorems are used to prove that such equilibria exist. For example, proofs of the existence of Nash and Walrasian equilibriua proceed from these theorems, so we are questioning the foundations of neoclassical economics. Specifically. Papadimitriou (1994) has proved that the computational complexity of Sperner's lemma, which serves as a constructive route to a proof of Brouwer, is essentially NP complete, among the most difficult problems in all computer science. Therefore, unless an alternative mechanism is given, social systems that depend solely on Brouwer or Kakutani should be viewed as unlikely, at best, to be realized in practice.

5.2 Rationality Postulate Derivative of Homogeneity Assumptions

Homogeneous rules of behavior—essentially utility maximization discounted over time or 'best reply' when agents are myopic—constitute the norm in economic modeling today. Assuming all people have the same decision-making rules is clearly false empirically (Newell and Simon 1972). Here we will argue this assumption is necessary to render rational models consistent, and that as soon as

[13] Indeed, any claim of the *necessity* of agent-level equilibrium for macro-equilibrium is guilty of the classical *fallacy of division* (Angeles 1981).

significant behavioral heterogeneity is introduced, rationality ceases to be of much use.

When heterogeneous behaviors are present—say, some largely rational, some merely purposive—then some behaviors may be able to prey upon others. But such predator-prey interactions are dynamic and the social environment can shift in such a way that roles get reversed and prey become predators. Otherwise rational agents, if they are even well-defined in such circumstances, may not be optimal—at least over particular epochs—given sufficient behavioral heterogeneity. For example, a simple 'best reply' strategy may be able to outperform a purely rational one in rapidly changing environments, precisely because it is adaptive and opportunistic. Diverse behaviors in large populations can perhaps best be thought of as *ecologies* of behaviors whereby action *A* lives off *B* which profits from interacting with *C* which can then reliably win against *A*, say.[14] Such behavioral ecologies can be stationary at the aggregate level despite constant flux at the micro-level, i.e., perpetual adaptation by agents.

An example of just this kind behavior in agent-based models is Lux (1997) in which there are various strategies for trading in an artificial financial market. The ability of fundamentalist (rational) traders to change to chartists plays an important role in the achievement of empirically significant price dynamics.

When economic models feature only rational agents then there do not exist such stable ecologies of behaviors to exploit and we are back in the world of aggregate social stability resulting from agent level (Nash) equilibrium. There are today a variety of models in which rational and non-rational agents co-exist with both populations capable of surviving over long time scales.

Departing from rationality brings another implicit operating principle of agent-based modeling into full view. We are completely open to mathematical analysis of our models, especially when the analysis attempts to deal with the full generality of agent models as described formally above. But we essentially always endow our agents with heuristics and rules of thumb, never permitting them to be 'agent mathematicians' who are as capable mathematically as are we (Gigerenzer *et al.* 1999). Related to this is the idea that agents can credibly follow utility gradients, groping for welfare improvements, while from behavioral economics we know that they are largely incapable of elaborate deductions about the behavior of others, e.g., performing backward induction.

5.3 Interaction Networks

Humans interact through social networks. Typically, agents in economic models either interact with one another with equal probability or do not directly interact at

[14] In other domains such interacting strategies are known as hypercycles (e.g., Eigen and Winkler, 1992, and Padgett, 1997)

all but rather use global information about other agents' behavior in their decision-making.[15]

In agent models it is easy to reproduce the conventional modeling assumptions (global interactions) but it is not much more difficult to reproduce the kinds of interaction topologies that occur in reality. These often have the character of 'small worlds' (Watts 1999), and are conveniently synthesized as an intermediate form between regular graphs, like lattices, and random graphs. Too, when empirical data on social networks are available these are easily incorporated into agent-based computational models.

5.4 The Emergent Macroeconomy

The actual economy of any city, region or nation is a vast, sprawling, dynamic and adaptive system of relatively loose inter-firm networks and tighter intra-firm organizations, all interacting through markets. The size and annual growth of an economy depends crucially on a variety of poorly understood human behavioral characteristics, including consumer sentiment, producer demand forecasts and the resulting intermediate goods orders, response to interest and exchange rate changes, discounting behavior, and compliance with tax policy. The economy consists of banks, bond dealers, grocery stores, an army of the self-employed, the unemployed, factories, and entrepreneurs, among many, many other business entities. The economy possesses top down regulatory structures (e.g., the U.S. Federal Reserve System) but is as much a creature of bottom up emergence, resisting outright control. Real economies are also highly evolutionary, non-stationary on long time scales, growing in absolute terms and continually being revolutionized technologically. Leading firms in one generation are often in decline by the next, dominant industries at the start of a century are rarely around by the end. The pace of technological change is only exceeded by the speed at which speculations about the 'new era' are dashed on the shoals of the omnipresent business cycle, driven by waves of over investment.

The macroeconomy grows out of the myriad interactions of the economic agents that compose it, so it is more than a little methodologically unnerving to discover the lack of coherent microeconomic foundations for today's competing schools of macroeconomics. Of course, the mere fact that there are competing schools is an indication of the lack of such foundations. But while the desire to have such foundations is not debated, the character of them is.

Once we learn how to build the main components of a market economy with agents, it will be possible to jettison macroeconomic (mis)specifications completely and simply have macro statistics emerge from the interactions of large numbers of individual agents. This will be a new era for macroeconomics and in

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[15] More recently, local interactions have made their way into economics (Kirman 1995, Durlauf 1999).

Table 3 we suggest that, at least in terms of hardware, this day may not be too far away.

Table 3. Prospective growth of agent capabilities on single workstations

Model	Feasible by	Agents
Small country macroeconomy	2002-2004?	$O(10^7)$
Industrial country macroeconomy	2004-2006?	$O(10^8)$
U.S. macroeconomy	2006-2008?	$O(10^9)$
World economy	2008-2010?	$O(10^{10})$

Today we recognize a previous era's search for a literal 'fountain of youth' to be quioxitic: from the perspective of modern science it is reasonable to believe that it is unlikely to naturally exist. Perhaps the same is true of 'microfoundations for macroeconomics. For there must be many micro-specifications of agent behavior that are consistent with any extant macro-economy. Juxtapose the locations of the local video and convenience stores and the same macroeconomy will result. How different would things really be if the auto industry were in New Jersey and pharmaceuticals were in Michigan? One way to frame such discussions is through the physicists' notion of *universality*, in which certain details of dynamical processes can be neglected if one captures other essential, universal features. Different kinds of molecules have similar freezing dynamics although in general quite different structure. Macroeconomics as a science is really only possible if something like economic universality exists, e.g., if wide classes of economies have the same growth dynamics, independent of whether the video and dry cleaning store locations are swapped.

Underlying all this is the problem that we don't have a coherent understanding of *emergence*, either in economic contexts or in general (Nagel and Paczuski 1995). The way purposive agents interact to produce not merely a change in quantity but a change in quality is a key to understanding the evolution of the economy. Until we understand emergence we will not understand macroeconomcis. An analogous situation exists in physics and other sciences today (Laughlin and Pines 2000, Laughlin *et al.* 2000):

> We call this physics of the next century the study of complex adaptive matter. For better or worse we are now witnessing a transition from the science of the past, so intimately linked to reductionism, to the study of complex adaptive matter, firmly based in experiment, with its hope for providing a jumping-off point for new discoveries, new concepts, and new wisdom.

6 The Future of Economics as Agent Computing

The economics profession was an early adopter of high speed digital computation (Mirowski 2001). The early machines were adept at solving equations, particularly linear ones, and economics with its growing mathematical orientation was one of the few disciplines readily capable of rendering its formalisms in the new digital language.

For a long time computing was synonymous with mathematics and the solution of equations. Indeed, the most popular programming language of the era was an acronym for FORmula TRANslation. What is even more, computer science at the time focused nearly exclusively on programming, and questions concerning the most appropriate programming language for particular problems (e.g., assembler for writing operating systems, COBOL vs. RPG for business systems, etc.) together with efficient numerical algorithms, constituted the bulk of the subject matter studied.

As modern computer science has morphed from the study of programming languages and numerical analysis, and as the subjects being analyzed have moved well beyond the mere solution of equations, the economics profession has been in something of a time warp, largely failing to adjust to new circumstances, missing the opportunity to adopt powerful new technology. This paper has argued that there exist sufficient conditions for this situation to change, and that it is necessary to change in order for economics, specifically, and social science, generally, to harness modern computing power.

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